



Stability evaluation of 1 Ω and 10 k Ω standard resistors using a step-down method

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Abstract. The traceability chain of electrical resistance in Brazil relies upon standard resistors of 1 Ω and 10 k Ω maintained by the Electrical Standardization Metrology Laboratory (Lampe) at the Inmetro. These standard resistors are periodically calibrated at the Bureau International des Poids et Mesures in France to ensure traceability to the International System of Units. Transport can influence the properties of the resistors and change their values. Lampe checks resistors' changes before and after calibration at the BIPM. The highest differences tend to occur with the 1 Ω resistors. However, drift rate evaluation becomes arduous when the check results show small changes in the values – of about 0.10 $\mu\Omega/\Omega$ – due to the uncertainty components involved. In this work, the step-down is a procedure to measure the 1 Ω resistors through a 10 k Ω resistor. The comparison of results between Lampe and the Quantum Electrical Metrology Laboratory shows a relative difference of less than 0.05 $\mu\Omega/\Omega$. The results agreement allowed Lampe to confidently use the step-down method to evaluate 1 Ω resistors' drift.

1. Introduction

The Inmetro, through its Electrical Standardization Metrology Laboratory (Lampe), provides research centers, laboratories, industries, electric power companies, and universities with calibration services of standard resistors in direct current (dc) in the 1 Ω to 10 k Ω range using current comparison. Such calibrations are traceable to the International System of Units (SI) as the Lampe periodically calibrates its own 1 Ω and 10 k Ω standard resistors at the Bureau International des Poids et Mesures (BIPM) in France.

Not only well-known factors such as temperature [1][2], humidity [1], pressure [2], short-term drift [3], leakage currents [4][5], insulation resistance of the resistor terminals and connection cables [5], and low-frequency resistance noise due to resistor material and resistance range [6] can affect the calibration of standard resistors; the transport of resistors can also exert an influence, as it may change the resistor values.

Regarding transport behavior, Jones [7] studied environmental conditions that could affect six 1 Ω Thomas-type resistors. Thomas resistors values may exhibit hysteresis with temperature changes, and the drift rate changes for some months after the temperature change event. In two out of six Thomas resistors, significant changes in resistance values were measured – about $-0.065 \mu\Omega/\Omega$, after cycling from 20 $^{\circ}\text{C}$ to 25 $^{\circ}\text{C}$.



Lampe calibrates two Thomas resistors of $1\ \Omega$ and one ESI (Electro Scientific Industries) -type resistor of $10\ \text{k}\Omega$ at the BIPM. Lampe checks the resistor's changes before and after calibration at the BIPM. The highest differences tend to occur with the Thomas resistors. In 2015, due to budget constraints, Lampe sent the standard resistors to the BIPM using airplane luggage and included a temperature data logger accompanying the resistors during the trip. It recorded temperature oscillations between $8\ ^\circ\text{C}$ and $30\ ^\circ\text{C}$. The value of one of the Thomas resistors changed at about $-0.4\ \mu\Omega/\Omega$, and Lampe suspended calibrations with this standard until the resistor value became stable again.

The interval for checking the resistors after the trip to the BIPM is six months or more. Typically, Lampe uses a set of five $1\ \Omega$ Thomas resistors to check the two calibrated $1\ \Omega$ Thomas resistors and a set of three $10\ \text{k}\Omega$ that includes ESI, Leeds, and Northrup (L&N) and Fluke type resistors to check the calibrated $10\ \text{k}\Omega$ ESI resistor. Environmental conditions are well controlled, and the method to check the $10\ \text{k}\Omega$ resistors using the 1:1 ratio is satisfactory. However, in checking Thomas resistors, drift rate evaluation becomes arduous when the results show small changes in the values due to the uncertainty components involved. Evaluating the hysteresis influence on Thomas resistors values is affected by the difference between the temperature coefficients of the resistors, the uncertainty of the bridge used in the measurements, and the six-month drift of the Thomas resistors. The monthly calibration routine of the Lampe hinders a more frequent follow-up of this drift.

As an alternative to this issue, the quantum Hall system (QHS) of the Inmetro, operated by the Quantum Electrical Metrology Laboratory (Lameq), was employed. Although not officially providing calibration services yet, the Inmetro has already sent the request for the insertion of new, QHS-based services in its Calibration and Measurement (CMC) capabilities listed in Appendix C of Mutual Recognition Organization of the International Committee for Weights and Measures (CIPM) – named CIPM MRA [8], since its QHS obtained good results in the BIPM.EM-K13.a&b bilateral comparison with the BIPM in 2022 [9]. Such good results enabled Lampe to use the QHS to validate the step-down method presented in this paper for evaluating Thomas resistors between calibrations.

2. Materials and methods

In this work, the step-down is a procedure to measure a $1\ \Omega$ resistor through a $10\ \text{k}\Omega$ resistor. The three resistors calibrated in the BIPM are coded as TH1 ($1\ \Omega$), TH2 ($1\ \Omega$), and R1 ($10\ \text{k}\Omega$). In this work, resistors TH1 and TH2 were measured with resistor R1.

Resistor R1 is the most stable among all resistors in Lampe and has a steady drift rate (of around $+0.08\ \mu\Omega/\text{year}$). This resistor was less influenced in transport compared to resistors TH1 and TH2.

The main parameters that influence the step-down are the temperature stability in the oil bath and air bath, leakage resistance of the resistors and their connection cables, uncertainty and stability of the bridge, temperature coefficients of the resistors involved, and the power dissipated by the resistor during measurements. The bridge used is a commercial bridge by Measurements International (MIL) model 6010D.

Due to a fault in its oil bath, Lameq could not perform QHS measurements on resistors TH1 and TH2. Nevertheless, Lameq has standard resistors in the range of $1\ \Omega$ to $10\ \text{k}\Omega$ with low first-order temperature coefficients (less than $0.5 \times 10^{-6}\ \Omega\ ^\circ\text{C}^{-1}$) that were fit to this work. These resistors are preferably measured in an oil bath but can be measured in an air bath.

The following procedures were adopted for better reliability: a) step-down of three $1\ \Omega$ resistors from the $10\ \text{k}\Omega$ resistor (R1) replacing Lampe resistors with Lameq resistors where necessary; b) QHS measurement of Lameq and Lampe resistors when possible; c) comparison of Lampe's results with those of Lameq.

During the step-down, the oil and air baths involved were maintained at a controlled temperature of $(23.000 \pm 0.004)\ ^\circ\text{C}$ and $(23.00 \pm 0.06)\ ^\circ\text{C}$, respectively.

3. Step-down procedure description and results

Lampe reduced, when possible, the main parameters that influence the measurements. The cables used to connect the resistors had conductor-to-conductor insulation resistance values greater than 1 TΩ for resistors connected in an air bath and values greater than 100 MΩ for resistors connected in an oil bath [10]. On the 6010D bridge, this range of values can cause an error of up to 0.01 μΩ/Ω for 10 kΩ resistors measured in the air bath and 0.1 μΩ/Ω for 10 kΩ resistors measured in the oil bath. For values smaller than 1 kΩ, the errors caused in the oil bath can reach up to 0.01 μΩ/Ω. Thus, to prevent these errors from influencing the step-down, 10 kΩ resistors were used only in the air bath. Furthermore, measurements were performed over one week to minimize the influence of the stability of the 6010D bridge, air bath, oil bath, and environmental conditions.

The ideal scenario is to use stable resistors in pairs, with temperature coefficients alpha (α_{23}) and beta (β) smaller than $0.5 \times 10^{-6} \Omega \text{ } ^\circ\text{C}^{-1}$ and $0.5 \times 10^{-6} \Omega \text{ } ^\circ\text{C}^{-2}$ (in absolute values). However, this was not the case since not all available resistors met these conditions. Table 1 shows α_{23} and β values for the resistors used.

Table 1. Temperature coefficients of the standard resistors used in the step-down.

Resistor code / (laboratory)	Manufacturer	Model	Nominal value	α_{23} ($10^{-6} \Omega \text{ } ^\circ\text{C}^{-1}$)	β ($10^{-6} \Omega \text{ } ^\circ\text{C}^{-2}$)
TH1 (Lampe)	L&N	4210	1 Ω	4.7	-0.5
TH2 (Lampe)	L&N	4210	1 Ω	4.272	-0.508
PT16 (Lameq)	Tinsley	5685A	1 Ω	-0.2103	-0.0161
6A (Lampe)	L&N	4025-B	10 Ω	1.0	-0.5
PT17 (Lameq)	Tinsley	5685A	10 Ω	0.379	-0.0246
7A (Lampe)	L&N	4030-B	100 Ω	6.0	-0.5
7D (Lameq)	L&N	SR-102/DC	100 Ω	-0.079	-0.019
7E (Lameq)	L&N	5685A	100 Ω	0.4	-0.07
8B (Lampe)	L&N	4035-B	1 kΩ	8.0	-0.5
PT18 (Lameq)	Tinsley	5685B	1 kΩ	0.4986	-0.0023
R1 (Lampe)	ESI	SR-104	10 kΩ	-0.11	-0.024
R3 (Lampe)	ESI	SR-104	10 kΩ	-0.07	-0.0255
9F (Lampe)	Fluke	742A-10k	10 kΩ	0.02	0.003

In table 1, it is possible to observe varied α_{23} values. Thus, finding a current that generated the lowest heat in the resistor and did not create instability in the 6010D bridge was required. Besides, at each range change, it was necessary to check the stability of the 6010D bridge through a triangulation between three resistors.

During the calibration performed with the 6010D bridge in the 10:1 configuration (Rx:Rs), the power P_s dissipated in the R_s resistor is 10 times greater than in the R_x resistor (R_x and R_s are the bridge terminals where the resistors are connected). The applied current in R_x was configured so that the dissipated power in R_s was less than 2.5 mW. Resistor 8B, whose α_{23} is $8.0 \times 10^{-6} \Omega \text{ } ^\circ\text{C}^{-1}$, showed a temperature increase of 0.006 °C when dissipating 2.5 mW, while the others had a maximum increase in temperature of 0.003 °C.

The temperature measured by the platinum thermometer in the resistor reflects a sample of the average temperature dissipated by the resistor core. Although this temperature value is enough to carry out the calibration of client resistors in Lampe, in step-down this temperature can have a significant influence and generate systematic errors during the change of steps. An error of 0.003 °C in the temperature measurement of the 8B can cause a relative error of 0.024 μΩ/Ω. To reduce this error, the maximum current applied to the 1 kΩ resistors 8B and PT18 was 1 mA and the power dissipated was about 1 mW.

Figure 1 shows the simplified diagram of the configuration used to carry out the measurements during the step-down. The current value indicated next to the arrow corresponds to the current value applied to the pointed resistor. The double line with a single arrow illustrates that in each cycle two measurements are performed. The double horizontal line with a double arrow indicates the measurement result corresponds to the mean in the forward and reverse directions.

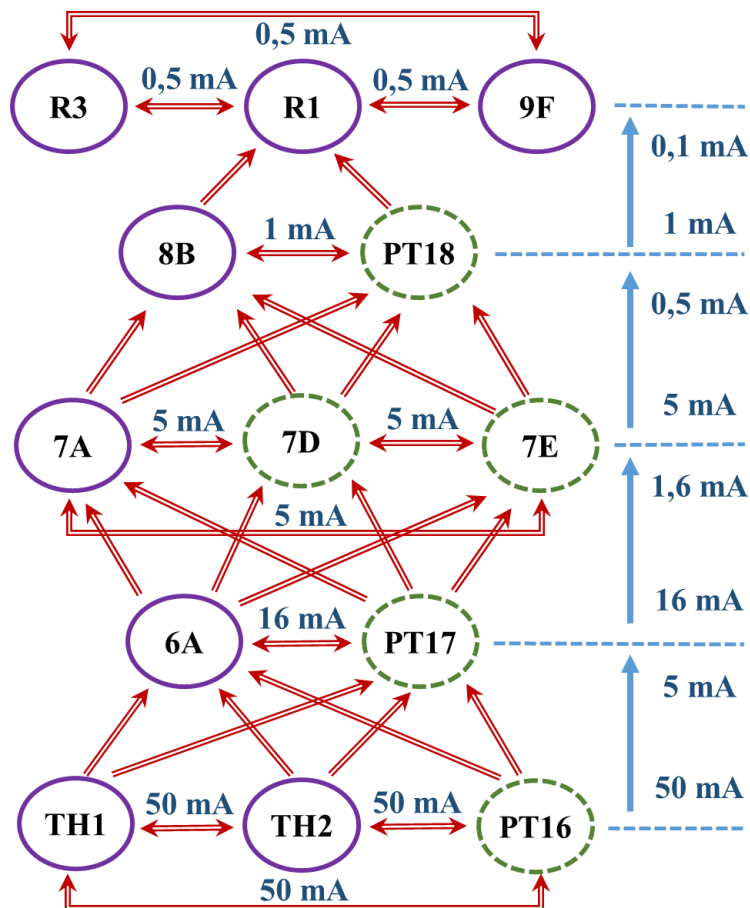


Figure 1. Simplified diagram of the configuration used to measure the resistors during the step-down. Solid purple line circles represent Lampe's resistors. Dashed green line circles indicate Lameq's resistors.

Although the 1Ω resistors are measured from the $10 \text{ k}\Omega$ resistor, the ratio measurements start from the 1Ω resistors to reduce the thermal effects during the step-down. Each 6010D program records the measured ratio between the resistors. From these records, it is possible to determine the values of all resistors as a function of $R1$'s value.

The sequence begins by performing a triangulation between three 1Ω resistors that dissipate 2.5 mW of power. Afterward, the measurement of the 10Ω resistor is carried out, maintaining the same current previously applied in R_s , whereas in R_x a power P_x ten times lower is obtained. Then, the power dissipated in the 10Ω resistor is increased to 2.5 mW , the resistor is pre-heated, and the triangulation between the 10Ω and 1Ω resistors is completed. This procedure is followed until reaching the $1 \text{ k}\Omega$ resistors. In the $1 \text{ k}\Omega$ resistors, a power of 1 mW is applied and in the $10 \text{ k}\Omega$, 0.1 mW . The step-down is finished with the triangulation in $10 \text{ k}\Omega$ of $R1$, $R3$ and $9F$ using a power of 2.5 mW . Table 2 shows the sequence used to perform these measurements.

Table 2. Sequence and configuration of measurements performed on the 6010D bridge.

Step	R_s	R_x	Applied current [$I_s:I_x$] mA	[$P_s:P_x$] mW	Resistors	6010D program
1 Ω triangulation	1 Ω	1 Ω	50:50	2.5:2.5	TH1, TH2, PT16	M1 to M12
Step 10:1	1 Ω	10 Ω	50:5	2.5:0.25	(TH1, TH2, PT16)- (6A, PT17)	M13 to M20
Step 1:1	10 Ω	10 Ω	16:16	2.5:2.5	6A, PT17	M21, M22
Step 10:1	10 Ω	100 Ω	16:1.6	2.5:0.25	(6A, PT17)-(7A, 7D, 7E)	M23 to M28
Step 1:1	100 Ω	100 Ω	5:5	2.5:2.5	7A, 7D, 7E	M29 to M34
Step 10:1	100 Ω	1 k Ω	5:0.5	2.5:0.25	(7A, 7D, 7E)-(PT18, 8B)	M35 to M40
Step 1:1	1 k Ω	1 k Ω	1:1	1:1	8B, PT18	M41, M42
Step 10:1	1 k Ω	10 k Ω	1:0.1	1:0.1	(8B, PT18)-(R1, R3, 9F)	M43 to M48
10 k Ω triangulation	10 k Ω	10 k Ω	0.5:0.5	2.5:2.5	R1, R3, 9F	M49 to M54

Table 3 shows the average temperature values measured in the resistors during the measurements. Resistors R1, R3, 9F, and 7D were kept in the air bath at an average temperature of 23.00 °C.

Table 3. Resistors mean temperature in oil bath.

Resistor code / (laboratory)	Dissipated power		
	≤ 0.25 mW	1 mW	2.5 mW
TH1 (Lampe)	-	-	23.0060 °C
TH2 (Lampe)	-	-	23.0045 °C
PT16(Lameq)	-	-	23.0055 °C
6A (Lampe)	23.0015 °C	-	23.0050 °C
PT17(Lameq)	23.0045 °C	-	23.0045 °C
7A (Lampe)	23.0055 °C	-	23.0090 °C
7E(Lameq)	23.0025 °C	-	23.0025 °C
8B (Lampe)	23.0045 °C	23.0070 °C	-
PT18(Lameq)	23.0035 °C	23.0035 °C	-

After the measurements, the values of all resistors were calculated from the predicted value of R1 on the measurement date, using the ratio values previously recorded by the 6010D programs. The values obtained from the measurements are shown in tables 4 and 5. U is relative expanded uncertainty, k is the coverage factor, and ν_{eff} denotes the effective degrees of freedom. The tables also have simplified results of the triangulations in each step, presented as “Dif” (relative difference). Through Dif, it is possible to evaluate the bridge stability and the temperature corrections of the resistors in the calibrations.

The term “Mean” used in tables 4 and 5 refers to the average between the results obtained through the resistors used as a standard. The correlation during measurements is high, and the difference between the results is much smaller than their uncertainty. Thus, to simplify the calculations, the uncertainty of the mean was considered the highest-valued term among the two used in calculating the mean.

The results of these triangulations show Dif between $-0.045 \mu\Omega/\Omega$ and $+0.034 \mu\Omega/\Omega$. The bridge uncertainty is $0.04 \mu\Omega/\Omega$. Dif values greater than the bridge uncertainty occurred due to the bath stability and the temperature coefficient of the resistors.

Table 4. Values obtained through the step-down between the 10 k Ω and 100 Ω resistors.

Rx	Rs	Measured value (Ω)	U ($\mu\Omega/\Omega$)	k	v_{eff}	Date (mm/dd/yy)
R1	Predicted	10000.01595	0.088	2.21	13.2	08/10/22
R3	R1	10000.02086	0.13	2.03	74.1	08/10/22
9F	R1	10000.06359	0.14	2.03	77.2	08/10/22
9F	R3	10000.06356	0.16	2.01	195	08/10/22
		Dif[9F(R3)-9F]	-0.003 $\mu\Omega/\Omega$			
8B	R1	1000.024687	0.12	2.07	38.5	08/07/22
PT18	R1	999.997651	0.10	2.13	20.5	08/07/22
PT18	8B	999.997659	0.12	2.05	47.1	08/07/22
		Dif[8B(PT18)-8B]	0.008 $\mu\Omega/\Omega$			
7A	Mean(8B;PT18)	100.001964 6	0.13	2.05	49.6	08/07/22
7D	Mean((8B;PT18)	100.0002099	0.15	2.02	111.2	08/07/22
7E	Mean(8B;PT18)	99.9999177	0.14	2.04	69.3	08/07/22
7D	7A	100.0002121	0.16	2.02	110.9	08/07/22
7E	7A	99.9999185	0.14	2.04	71.4	08/07/22
7E	7D	99.9999211	0.16	2.02	129.2	08/07/22
		Dif[7D(7A)-7D]	0.022 $\mu\Omega/\Omega$			
		Dif[7E(7A)-7E]	0.008 $\mu\Omega/\Omega$			
		Dif[7E(7D)-7E]	0.034 $\mu\Omega/\Omega$			

Table 5. Values obtained through the step-down between the 100 Ω and 1 Ω resistors.

Rx	Rs	Measured value (Ω)	U ($\mu\Omega/\Omega$)	k	v_{eff}	Date (mm/dd/yy)
6A	Mean(7A;7D;7E)	9.99997239	0.16	2.02	129.8	08/07/22
PT17	Mean(7A;7D;7E)	10.00000647	0.16	2.02	129.5	08/07/22
PT17	6A	10.00000619	0.16	2.02	147.1	08/07/22
		Dif[6A(PT17)-6A]	-0.028 $\mu\Omega/\Omega$			
TH1	Mean(6A;PT17)	0.999992173	0.17	2.02	154.9	08/07/22
TH2	Mean(6A;PT17)	0.999993152	0.17	2.02	153.0	08/07/22
PT16	Mean(6A;PT17)	1.000001890	0.17	2.02	147.7	08/07/22
TH2	TH1	0.999993135	0.17	2.01	173.9	08/07/22
PT16	TH1	1.000001846	0.18	2.01	187.8	08/07/22
PT16	TH2	1.000001845	0.18	2.01	182.9	08/07/22
		Dif[TH2(TH1)-TH2]	-0.017 $\mu\Omega/\Omega$			
		Dif[PT16(TH1)-PT16]	-0.044 $\mu\Omega/\Omega$			
		Dif[PT16(TH2)-PT16]	-0.045 $\mu\Omega/\Omega$			

4. Measurement results with the quantum Hall system

The measurements carried out by Lameq in the air bath obtained satisfactory results and allowed intermediate comparisons to the results by Lampe. This way, it was possible to reevaluate the values of resistors TH1 and TH2 with greater reliability. Table 6 shows the results of measurements performed on Lameq.

Table 6. Results of Lameq measurements.

Rx	Rs	Measured value (Ω)	U ($\mu\Omega/\Omega$)	k	v_{eff}	Date (mm/dd/yy)
R1	7D	10000.015358	0.0098	2.01	323.8	09/05/22
R3	7D	10000.02051	0.010	2.01	193.8	09/08/22
9F	7D	10000.06319	0.0098	2.00	∞	09/13/22
PT18	7D	999.9976347	0.011	2.00	∞	09/26/22
7D	QHS2	100.0002052	0.011	2.52	6.6	08/28/22
7E	7D	99.9999107	0.011	2.32	9.3	09/15/22
PT17	7D	10.00000549	0.012	2.17	16.5	09/24/22
PT16	PT17	1.000001785	0.014	2.06	41.4	09/30/22

5. Comparison of results between Lampe and Lameq

The differences between the results obtained by Lampe and Lameq and the absolute values of normalized error ($|En|$) are shown in table 7. The values obtained by Lampe were based on the predicted value of R1. The interval time between resistor measurements of Lampe and Lameq was about 45 days. It was necessary to consider the monthly drift rate of the resistors (table 8) to evaluate the Dif. Still, when the monthly drift values are very close to the uncertainty of the 6010D bridge, which is $0.04 \mu\Omega/\Omega$, it is necessary to verify the results of the triangulation of the resistors during the step-down.

Table 7. Summary of Lampe and Lameq results.

Resistor	Lampe		Lameq		Dif ($\mu\Omega/\Omega$)	$ En $
	Measured value (Ω)	U ($\mu\Omega/\Omega$)	Measured value (Ω)	U ($\mu\Omega/\Omega$)		
R1	10000.015950	0.088	10000.015358	0.0098	-0.0592	0.67
R3	10000.02086	0.13	10000.02051	0.010	-0.035	0.27
9F	10000.06359	0.14	10000.06319	0.0098	-0.040	0.29
PT18	999.9976510	0.10	999.9976347	0.011	-0.016	0.16
7D	100.0002099	0.15	100.0002052	0.011	-0.047	0.31
7E	99.9999177	0.14	99.9999107	0.011	-0.070	0.50
PT17	10.0000647	0.16	10.00000549	0.012	-0.098	0.61
PT16	1.000001890	0.17	1.000001785	0.014	-0.104	0.61

Table 8. Monthly drift of resistors measured at Lameq.

Resistor	Monthly drift ($\mu\Omega/\text{month}$)
R1	0.007
R3	0.010
9F	0.016
PT18	0.012
7D	0.038
7E	0.006
PT17	0.007
PT16	0.011

Table 7 shows that the $|En|$ between the measurements carried out between Lampe and Lameq are satisfactory, that is, $|En| < 1$, and agree with Lampe's CMC. If the predicted value of R1 is corrected with the value measured in Lameq and the monthly drift of R1, all resistors $|Dif|$ drops to a value less than $0.05 \mu\Omega/\Omega$.

The comparison of values between Lampe and Lameq shows that it is possible to evaluate the tendency of resistors TH1 and TH2 from the step-down of R1 resistor.

6. Conclusion

Despite the difficulties encountered in calibrating the oil resistors in the air bath by Lameq, the results were positive and increased the reliability of the step-down method to verify the Lampe resistors calibrated at the BIPM.

This comparison permitted Lampe to identify the alterations that occurred during the transport of resistors TH1, TH2, and R1 to/from the BIPM and to reevaluate the points used in the calibration history to predict R1's value.

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References

- [1] Braudaway D W 1999 *IEEE Trans. Trans. Instrum. Meas.* **48**(5) 878–882
- [2] Jones G and Elmquist R E 2007 *NCSL Int. Meas.* **2** 42–48
- [3] Giblin S P and Drung D 2018 Limitations to the Short-Term Stability of 100 M Ω and 1 G Ω Standard Resistors *Proc. 2018 Conference on Precision Electromagnetic Measurements (CPEM 2018) (Paris)*. DOI: 10.1109/CPEM.2018.8501095
- [4] Cabral V, Ribeiro L and Sousaet J A 2018 *J. Phys.: Conf. Ser.* **1044** 012073
- [5] Honig R 2010 *Cal Lab Magazine* **17**(1) 22–28 Available at <https://www.callabmag.com/wp-content/uploads/2011/02/jan10.pdf>
- [6] Silva M C 2019 *Projectus* **4**(1) 88–101. DOI: 10.15202/25254146.2019v4n1p88
- [7] Jones G R, Pritchard B J and Elmquist R E 2009 *Metrologia* **46**(5) 503–511
- [8] Rolland B et al 2022 *Metrologia* **59**(1A) 01004
- [9] Comité International des Poids et Mesures 1999 *Mutual recognition of national measurement standards and of calibration and measurement certificates issued by national metrology institutes* (Paris, France: BIPM)
- [10] Silva M C and Geronymo G M 2021 Evaluation of the effects of leakage current in the calibration of standard resistors between two dc resistance bridges and a digital sampling impedance bridge *Proc. XIV Congresso Internacional de Metrologia Elétrica (SEMETRO) (Rio de Janeiro)* 204823 Available at <https://metrologia2021.org.br/?p=2304>