Ramsey pulse optimization for atomic fountains

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Abstract. Atomic frequency standards realize the measurement of the atomic transition frequency between two well-defined quantum energy levels. This study uses the Euler-Cromer method to numerically analyze the atomic transition probabilities for the cesium fountain clock, based on the Ramsey method of separate fields.Consequently, this examination successfully attains an optimal compromise correlation among the oscillating field (B_O), static field (B_Z), and detuning (δw). Thus, this work presents a new tool for time and frequency metrology standardization.

Keywords: Rabi oscillations; Ramsey pulse; two-level system; cesium fountain clock; time and frequency metrology

1. Introduction

Understanding a two-level quantum system is vital in many areas, such as quantum computation [1], quantum metrology [2], quantum sensing [3], and quantum frequency standardization [4]. For this last application, cesium fountain clocks serve as reference to the international atomic time scale TAI [5]. The operating mode of cesium atomic clocks uses the microwave cavity to induce atomic transitions between hyperfine two-level states of the cesium atom. This type of cesium clock presents exceptional levels of uncertainty measurement with stability on the order of 10^{-15} $\tau^{-1/2}$ and an accuracy estimated for a relative uncertainty of 10^{-16}.

The operation of the cesium fountain clock is based on Rabi oscillations [6] and the Ramsey method [7], in which an atomic cloud is interrogated twice through a resonator (microwave cavity), generating the atomic transition. The Rabi and Ramsey signals are fundamentals for performing a cesium fountain clock. Many factors can affect the atomic transition in this type of clock, like gravitational effects [8], AC Stark effect [9], Zeeman effect [10], black body radiation [11], and others. All these effects degrade the signal, causing undesirable frequency shifts.

In particular, in an atomic cesium fountain clock, a static magnetic field named C-Field, is applied to the region from below the microwave cavity to the highest point the atoms reach to perform the Ramsey method during the time of flight of the atoms. The interaction of the atoms with the C-field is used to provide a quantization axis to separate the field-dependent transitions from the clock transition.$6^2S_{1/2}$ $|F = 3, m_f = 0\rangle \rightarrow 6^2P_{1/2}$ $|F = 4, m_f = 0\rangle$, to avoid overlapping lines as much as possible [10]. However, C-field induces a significant frequency shift of the measured hyperfine levels of the atoms that need to be corrected for better clock performance. In the scientific literature, this frequency shift is known as a Second-Order Zeeman Shift (SOZS) [10, 12, 13]. One way to estimate the SOZS is to obtain averaged C-field values from Zeeman frequency data for several launching heights of the atomic cloud [14, 15].

In this context, the present work proposes a numerical solution based on the Euler-Cromer method for studied Ramsey pulses used in the clock of atomic cesium fountain. Using this new computational tool, an optimization can be made for the value of the oscillating field coming from the microwave cavity, the C-field for all interaction regions of the atoms, and an estimation for the detuning value is performed. Our goal is to evaluate the impact of the C-field on the atoms, obtaining a suitable time interval that minimizes the detuning. Therefore, this work aims to contribute to both the application fields in quantum metrology and time and frequency metrology.

2. Basic principle

2.1. Theorical model

The atomic transition between two hyperfine levels of the states $6^2S_{1/2}$ $|F = 3, m_f = 0\rangle \rightarrow 6^2P_{1/2}$ $|F = 4, m_f = 0\rangle$, reproduces the primary definition for the second, according to the new SI. To excite an atomic transition between these levels in the cesium fountain clock, the Ramsey interrogation method [7] is used, in which the atoms undergo two oscillatory perturbations separated by a perturbation-free region. The atoms interact with the C-field throughout the process, causing the frequency shift. As the magnetic induction separates each of the seven transitions $\Delta F = \pm 1$ e $\Delta mf = 0$ of their neighborhoods, approximating a two-level system interacting with magnetic fields is valid. A basic schematic diagram of the cesium fountain is shown in Figure 1.

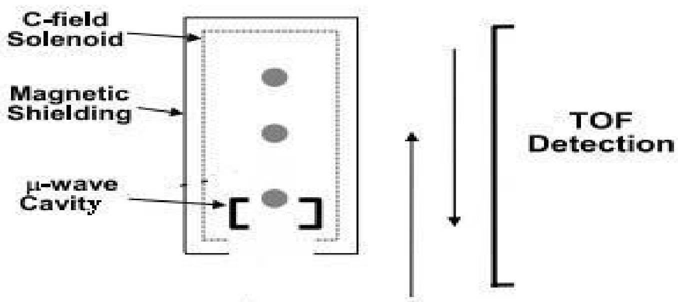

Figure 1: A schematic diagram of the cesium fountain clock.

As can be seen in Figure 1, the flight tower consists of a microwave cavity for interrogating the atomic cloud, a solenoid to produce the C-field, and a magnetic shield to prevent perturbations from stray magnetic fields. A cylinder of μ-metal is placed around the free-flight tower [16]. For weak static and oscillating field values, it is possible to describe all the interactions experienced by the atomic cloud by:

$$
\begin{aligned}
i\hbar\dot{C}_3 &= \frac{g_j\mu_b\,B_Z}{2}C_3 + \frac{g_j\mu_b\,B_O}{2}e^{-i\delta wt}C_4 \\
i\hbar\dot{C}_4 &= -\frac{g_j\mu_b\,B_Z}{2}C_4 + \frac{g_j\mu_b\,B_O}{2}e^{i\delta wt}C_3
\end{aligned}
\tag{1}
$$

where ħ is the reduced Planck constant; g_j is the gyromagnetic factor of the electron; μ_b is the Bohr magneton; B_Z and B_O are the static and oscillating magnetic fields, respectively, and δw is the detuning between the frequency atomic transition and microwave frequency. C_3 and C_4 are the eigenvalues associated with probability amplitudes of the hyperfine levels [17, 18]. One can rewrite equation 1 as:

$$
\begin{aligned}
i\dot{C}_3 &= \frac{\gamma}{2}C_3 + \frac{b}{2}e^{-i\delta wt}C_4 \\
i\dot{C}_4 &= -\frac{\gamma}{2}C_4 + \frac{b}{2}e^{i\delta wt}C_3
\end{aligned}
\tag{2}
$$

where $\gamma = g_j\mu_B B_Z/\hbar$ is the Lamor frequency and $b = g_j\mu_B B_O/\hbar$ is the Rabi frequency.

2.2. *Numerical solution*

On this point, we describe the method for calculating the transition probability between the two levels, assuming the approach to calculate a single atomic transition probability [19]. Thus, applying these assumptions, the Euler-Cromer method is used to solve the coupled differential equations (2) [20], which can be written as:

$$
\begin{aligned}
C_{3i+1} &= -\frac{i}{\hbar}\left[\frac{\gamma}{2}C_{3i+1} + \frac{b}{2}e^{-i\delta wt}C_{4i+1}\right](t_{i+1} - t_i) + C_{3i} \\
C_{4i+1} &= -\frac{i}{\hbar}\left[\frac{\gamma}{2}C_{4i+1} + \frac{b}{2}e^{i\delta wt}C_{3i+1}\right](t_{i+1} - t_i) + C_{4i}
\end{aligned}
\tag{3}
$$

The programming language used in this study was the Python language. The sequence of coding the numerical solution using the Euler-Cromer method was given as follows: a) importing the libraries; b) defining the known parameters, with δw, B_Z and B_O; c) defining the initial conditions, namely $C_3 = 0 - 0j$, $C_4 = 1.0 + 0j$ and $t_i = 0$; d) defining the time step $dt = t_i + 1 - t_i = 1$ μs; e) implementing the Euler-Cromer method and ploting graphs for analysis. The coding of the Euler method was performed by repeating steps:

1) Optimization of the oscillating field B_O - The oscillating magnetic field B_O was characterized with $B_Z = 0$ and with detuning $\delta w = 0$. It evaluated typical values of the B_O used in the cesium fountain, on the order nano tesla. At this point, the condition b$\tau = \pi/2$ was inserted, where τ is the interaction time between the B_O and the atomic cloud. In addition, TOF was keeped of fixed at 20 ms.
2) Insertion and characterization of the static field B_z - We characterize the static magnetic field embedded in the entire trajectory of the atoms to realize the Ramsey method.
3) Insertion of the detuning δw - The detuning δw was inserted to compensate for the effect caused by the C-field again to achieve the maximum transition probability between hyperfine levels. However, to recover the $\pi/2$ pulse in the two passes through the microwave cavity, made a variation in the amplitude of the oscillatory field is to maximize the transition probability in the Ramsey method.

It is important to mention that the atomic cloud was considered as a single atom in this numerical simulation.

3. **Results**

The impact of different oscillating magnetic field B_O, static magnetic field B_z and detuning δw for a TOF of 20 ms is evaluated through Euler-Cromer method. The first result of this study is a graph of the behavior of the transition probability amplitudes as a function of the total time of the Ramsey process, in which, the oscillating magnetic field is characterized (Figure 2).

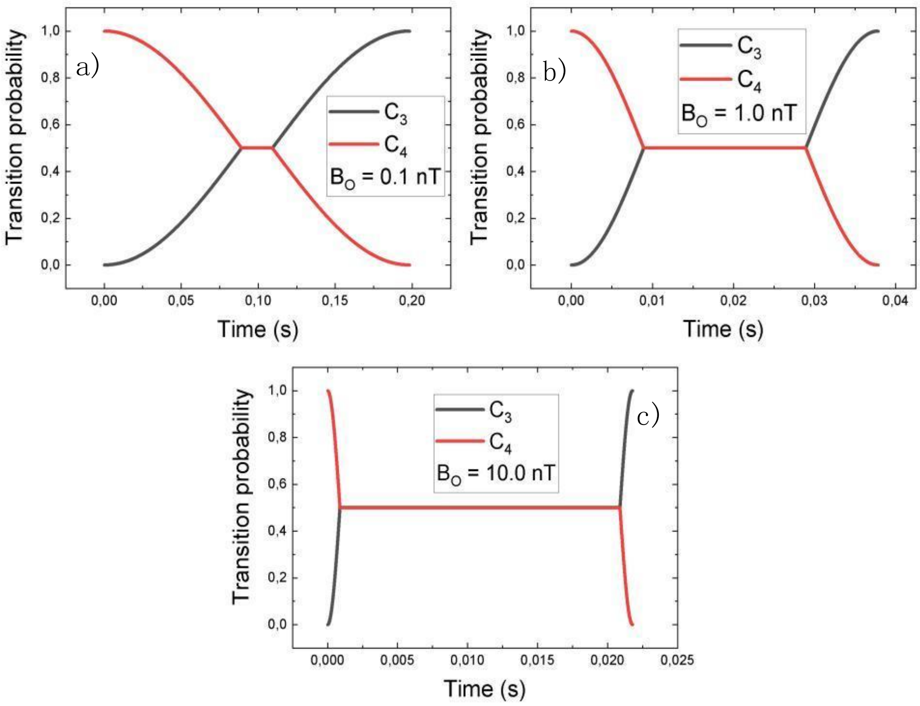

Figure 2: Transition probability amplitudes as a function of the total time of the Ramsey process to different oscillating magnetic fields B_O.

As can be seen in Figure 2, three typical values of the B_O used in a cesium fountain clock were applied. It can be seen that the interaction time τ is reduced as the value of the amplitude B_O

increases. From this first analysis, the value of the oscillating field was fixed at 1 nT (Figure 2b), corresponding to the value of τ = 8.92 ms. The next step is inserting the B_Z with an oscillating magnetic field fixed at 1.0 nT. Thus, we now have the spin precession phenomenon generated by the action of this field characterized by the Lamor frequency [17]. Figure 3 shows the effect of the static field on the transition probability between levels.

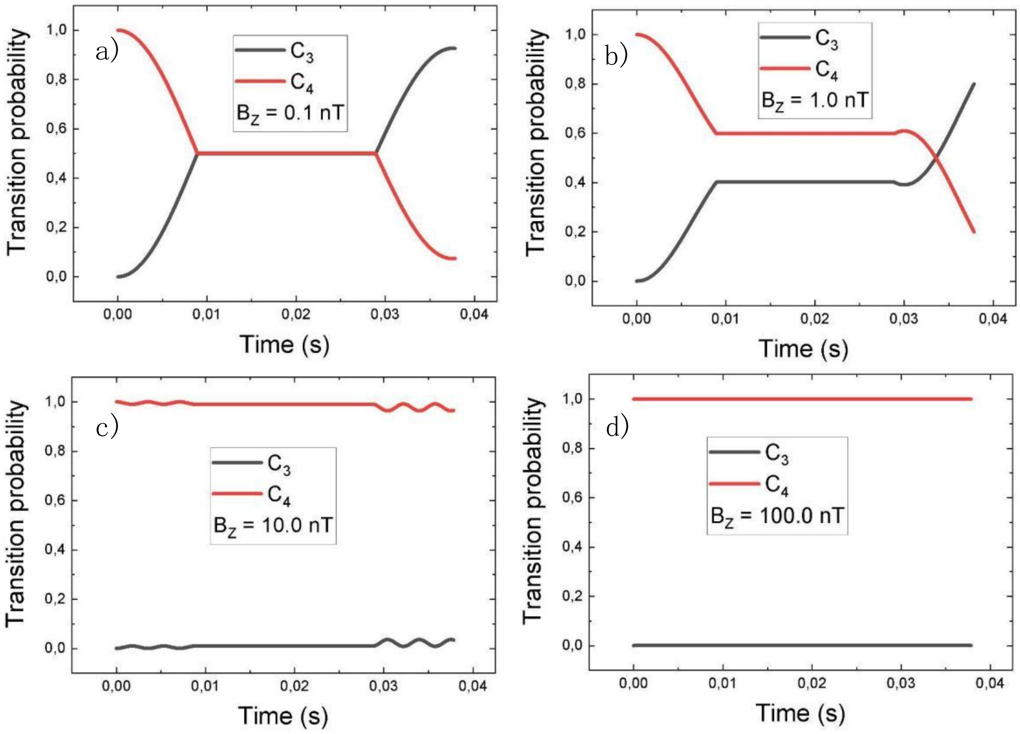

Figure 3: Transition probability amplitudes as a function of the total time of the Ramsey process to characterization static magnetic fields B_Z.

Figure 3 shows values that were used for characterizing the B_Z. It can be seen in Figure 3a that there is no more significant change in the transition amplitude on the first π/2 pulse. Figure 4b shows that the value of a static field of 1 nT is sufficient to generate the desired quantization on the axis of the atoms with field action, impairing the transition probability between the two π/2 pulses. Furthermore, it can be seen in Figure 4c and Figure 4d that for a static field value of the order of 10 nT and 100 nT, the transition probability is drastically reduced, and the transition between energy levels does not occur.

In a cesium fountain-type clock, typical C-Field values are on the order of hundreds of nano tesla [10]. Thus, the value of the B_Z was fixed at 100 nT, and a detuning δw was inserted, in addition to increasing the amplitude of the oscillating field B_0 focusing on recovering the two π/2 pulses, as can be seen in Figure 4.

Figure 4 expresses a compromise relation between the parameters varied to obtain the two π/2 pulses in the cesium fountain. Thus, by changing the value of the oscillating field, its interaction time with the atoms was reduced to τ = 0.44 ms.

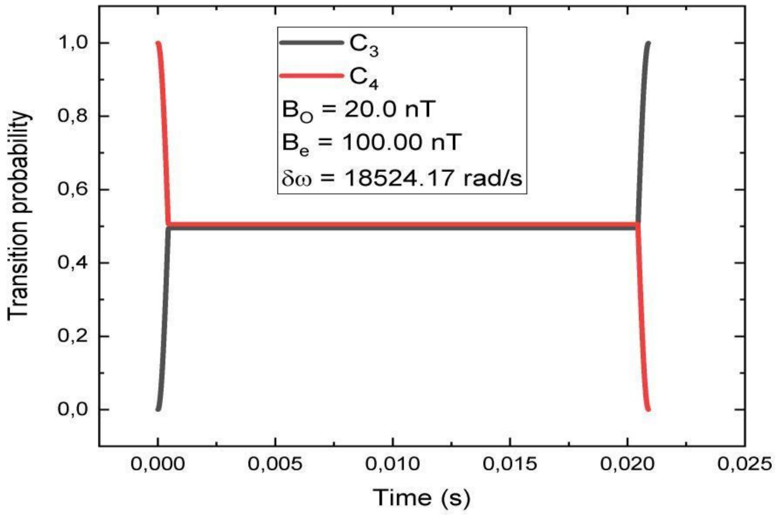

Figure 4: Amplitudes of transition probability as a function of total time of the Ramsey process to optimization of the parameters B_O, B_Z and δw.

4. Conclusion

The present work evaluates the impact of the static field insertion in a cesium fountain-type clock employing a numerical solution of the Rabi oscillations. The numerical method used in this study is Euler-Cromer. Focusing on evaluating were done by varying the input parameters oscillating magnetic field B_O, static magnetic field B_Z, and detuning δw. From the analysis performed, it was possible to obtain a compromise relation for the input parameters, in which, for B_O= 20.0 nT, Bz = 100 nT, and δw = 18,524.17 rad/s, correspondent to δf = 2,949.70 Hz. In addition, the interaction time of the atoms with the oscillating field was estimated through numerical simulation to be around 0.44 ms, for a value of TOF = 20 ms. Future work includes consideration of the inhomogeneities of the oscillating magnetic field and the static magnetic field.

5. Acknowledgment

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