Operating principle of electric current primary standard based on Ohm's law, Josephson and quantum hall effects

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Abstract. Ammeters in the range between nanoampere and miliampere are very used in fundamental and applied metrology, industry, microelectronics, medical care, and health. The calibration of these instruments needs calibration standards to provide a minimum quality process and traceability essential for measurement system. In Brazil, it does not exist a DC current primary standard based on Ohm´s law, which is one of BIPM recommendations to realize the ampere. In this ongoing work, it is being developed an electric current primary standard based on Ohm´s law in the range between nanoampere and microampere. This paper shows the most important phenomena present in electric current standards based on Ohm´s law obtained through the integration of a standard resistor, directly calibrated against a Quantum Hall System, and a Josephson Voltage Standard. Additionally, this paper presents the result of the software developed so far, in python language, to perform this integration of the system's components. As a result, it was obtained an understanding of the main quantum phenomena in these primary standards of voltage and electrical resistance. This understanding is fundamental for the development of electric current primary standard based on Ohm's law.

1. Introduction

The use of electrical current meters in the range between nanoamperes and milliamps is widely used in fundamental and applied metrology, in the semiconductor industry and health (dosimeters) [1]. The calibration of these meters requires calibration standards to provide the minimum quality of the process and the necessary traceability of the measurement system.

For the calibration of current meters, BIPM (2019) established 3 ways to do the practical realization of the ampere: by using Ohm's law, the unit relation $A = V/\Omega$, from practical realizations of voltage and resistance; by using a single electron transport (SET), the unit relation $A = C/s$; and by using the ratio $I = C\, dV/dt$, the unit relation A = F·V/s, which corresponds to the current flowing in a capacitor of capacitance C submitted to a voltage ramp over time. Although not established by the BIPM, there is still a fourth method, called “Ultra Low-noise Current Amplifier” (ULCA) which consists of the association of operational amplifiers and resistors. Worth mentioning that at the 26th General Conference on Weights and Measures (CGPM), held in 2018, there was a redefinition of the SI, that among other decisions, it established that a unit can be realized by any convenient equation of physics that links the defining constants to the quantity intended to be measured [2].

Brazil still lacks an electric current primary standard. This paper aims to clarify the working principles involved in the development of an electric current primary standard based on Ohm's law, which integrates the primary standards of voltage and electrical resistance which are obtained from the Josephson and Quantum Hall effects respectively. Additionally, it is presented the partial result of this integration, still under development.

The Josephson effect, discovered in 1962 by the physicist Brian David Josephson, consists of the observation of the following phenomenon: a junction, called Josephson junction (JJ), composed of two superconductors separated by an insulating material, will have a current $I(t)$, called critical current , between these superconductors through the insulator even if there is no application of an electric potential ($V_J = 0$) between these superconductors, as shown in figure 1 [3].

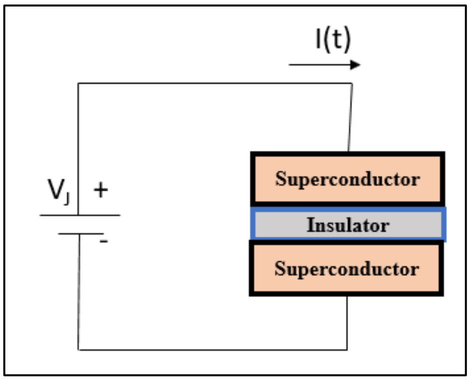

Figure 1. Josephson Junction representation submitted of a voltage V_J.

This current exists in both directions and occurs due to the Cooper pairs present in the superconductor and the quantum tunneling that allows these pairs to cross the insulator. This observation is called the Josephson DC effect and the critical current for zero JJ voltage is shown in figure 2 where, for $V_J = 0$, it is observed that the current I, presented on the ordinate axis, reaches values, greater and less than zero, which occupy up to the second division of the oscilloscope resolution. Such current is not result of Ohms's law, but tunneling effect, mainly related to Cooper pairs (characteristics of superconductor materials)[4].

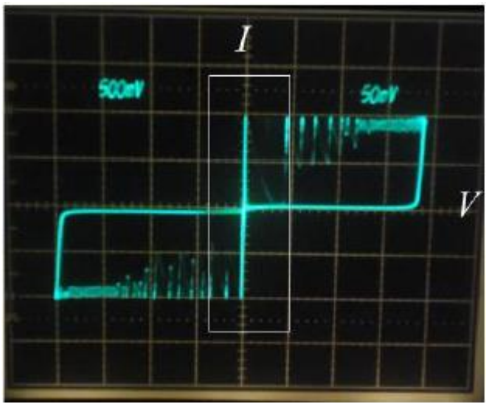

Figure 2. Current I that flows through the Josephson Junction as a function V_J.

Additionally, if in the same structure shown in figure 1 is applied a voltage V_J between the superconductors, a current with a well-defined frequency is observed and it frequency current depends on the potential difference V_J applied. This effect, called the ac Josephson effect, occurs due to the same phenomena described in the Josephson DC effect (Cooper pairs and quantum tunneling) and is shown in figure 2 which shows current values for a voltage a little less than 50 mV and a little less than -250 mV (500 mV scale) [5].

Thus, due ac Josephson effect, a direct relationship is established between the voltage at the terminals of a JJ and the frequency of the current flowing through the JJ's insulator, whose relationship is defined by equations (1) and (2). And so, when crossing a JJ with a current of frequency f, a voltage V_J that is obtained, in this context, depends only on the frequency of the current applied to JJ and its associated uncertainty [6].

$$f = \frac{2.e}{h} . V_{\mathrm{J}} = K_{\mathrm{J}} . V_J \tag{1}$$

$$K_J = \frac{2 \cdot e}{h} = 483\ 597\ 848\ ,416\ 984 \ \ GHz/V \tag{2}$$

Where:
f is the frequency of the current flowing through the junction;
K_J is the Josephson constant [2];
e is the elementary charge of the electron, given by $1.602\ 176\ 634\ .10^{-19}$ C [2];
h is Planck's constant, given by $6.626\ 070\ 15.10^{-34}$ J s [2];
V_J is the voltage applied across the terminals of the Josephson junction.

The Josephson effect is used in a primary voltage standardization system, not explored in this paper, called Programmable Josephson Voltage Standard (PJVS) [6]. Basically, this system receives a frequency signal around 75 GHz or 20 GHz and submits groups of JJ, called subarray, to this frequency signal which will induce the current I that will circulate through the JJ. As previously described in the ac Josephson effect, as a result of this current through the junction, between the terminals of a group of JJ will be observed voltage values V_J that, in certain regions, remain constant for small variations in polarization current, as shown in figure 3.

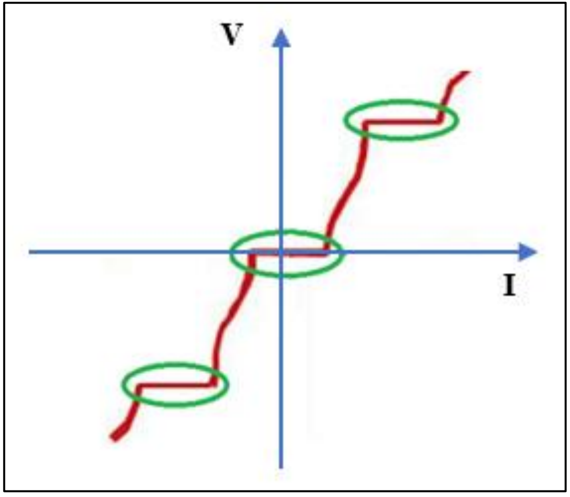

Figure 3. Voltage between the terminals of subarray subjected to bias current and microwave frequency around 20 GHz or 75 GHz.

And due the curve shown in figure 3, is possible to subject a group of JJ to an additional direct current (the one that is a consequence of the frequency of 20 GHz or 75 GHz) and thus mathematically determine through equation 1 and other equations not discussed, the Josephson voltage value V_J [5]. This Josephson voltage value V_J, resulting from the association of one or more JJ groups, makes it possible to calibrate an external voltage standard through a null detector.

The hall effect is observed a conducting plate, subjected to a current I and a magnetic field $\vec{B}$ perpendicular to this current, will have a magnetic force $\vec{F_m}$ induced due to the interaction between the charge velocity $-q$ and the magnetic field $\vec{B}$, given by equation (3) [7]. A representation of this phenomenon is shown in figure 4:

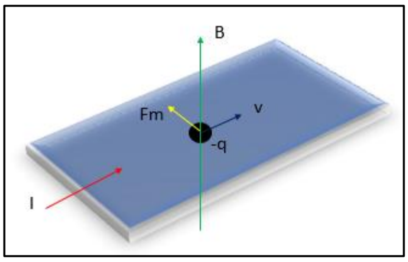

Figure 4. Magnetic force $\vec{F_m}$ acting on a charge $-q$ with velocity $\vec{v}$.

$$\vec{F_m} = q \cdot \vec{v} \times \vec{B} \tag{3}$$

Where:
$\vec{F_m}$ is the magnetic force, in N;
q is the charge value, in C;
$\vec{v}$ is the speed of the charge, in m/s;
$\vec{B}$ is the magnetic field in A/m .

Due to the magnetic force, the charges will concentrate on one side of the plate. This concentration creates a voltage V_H and an electric field $\vec{E}$ between each side of the plate (parallel to the velocity v). The interaction of the electric field $\vec{E}$ with the charge creates an electric force $\vec{F_{el}}$ that acts on the charge until it balances with the magnetic force $\vec{F_m}$. figure 5 shows the phenomenon described, called the Hall effect.

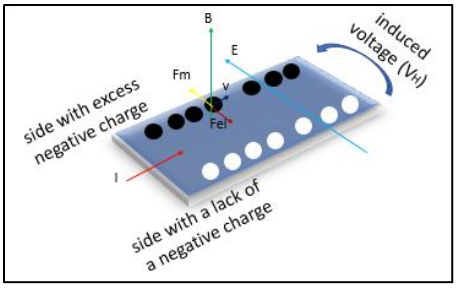

Figure 5. Hall effect representation.

According to Ohm's law, the relationship between the longitudinal electrical voltage (V_L) that generates the current I and this same current defines the longitudinal resistance, presented in equation (4) and the relationship between the electrical voltage originating from the electric field E (V_H) and the current I defines the Hall resistance R_H, presented in equation (5)[8].

$$R_L = \frac{V_L}{I} \tag{4}$$

$$R_H = \frac{V_\mathrm{H}}{I} \tag{5}$$

The quantum hall effect occurs at temperatures below 0.3 K, where quantum phenomena are observed, the curve of these resistances, as a function of the magnetic field $\vec{B}$, is shown in figure 6:

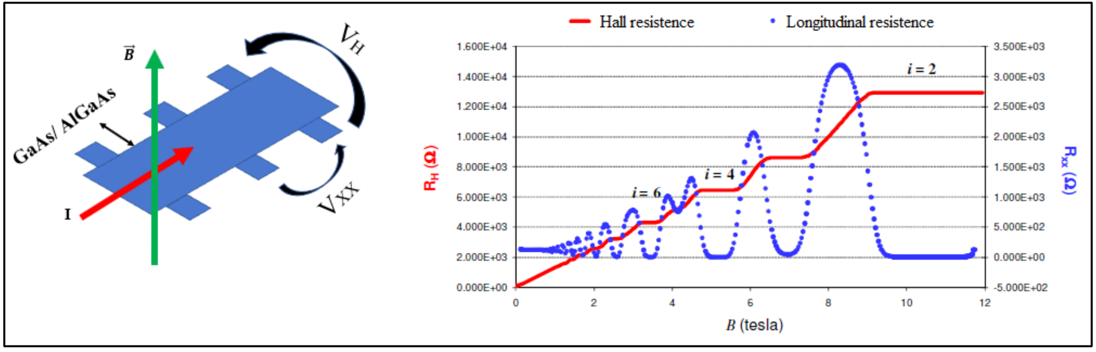

Figure 6. Representation of the electrical resistance curves as a function of the magnetic field.

In 1980 K. Von Klizing subjected the SI-MOSFET material to magnetic fields above 19.8 T and temperatures below 4.2 K. Thus, he determined the fundamental von Klitzing constant, R_K [9]. This constant depends on the elementary charge e and Planck's constant h, according to the equation (6) [10]:

$$R_K = \frac{h}{e^2} = 25\,812{,}807\,459\,3045\ \Omega \tag{6}$$

Where:
R_K is the von Klitzing constant [11];
h is Planck's constant, given by $6.626\,070\,15 \cdot 10^{-34}\ J\ s$ [2];
e is the elementary charge of the electron, given by $1.602\,176\,634 \cdot 10^{-19}\ C$ [2].

The resistance R_H can be expressed as a function of the constant R_K according to the equation (7) [9]:

$$R_H = \frac{R_K}{i} \tag{7}$$

Where i takes only integer values, and therefore R_H takes only values that are multiples of R_K. The i value represents the plateau of the magnetic field in the curve shown in figure 6.

In this way, the resistance standard is obtained by subjecting a semiconductor material (generally the GaAs and AlGaAs junction) to an intense magnetic field at temperatures below 4 K, as shown in the figure 7:

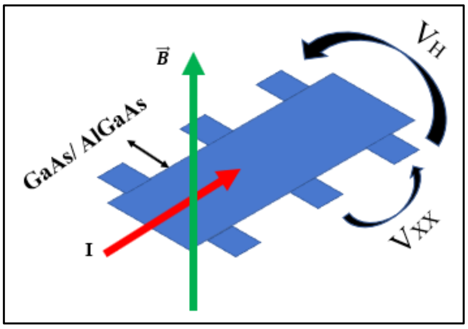

Figure 7. GaAs and AlGaAs junction subjected to a current I, magnetic field $\vec{B}$ at a temperature below 4 K.

Thus, when varying the magnetic field, the resistance value R_H (function of V_H) and resistance R_L (function of V_{XX}) are as shown in figure 6. The standard resistance value is then obtained from the identification of the plateau of the curve shown in figure 6 (i value) and dividing the R_K constant by the identified i [9].

Thus, typical 100 Ω standard R_S resistors are calibrated by comparing them with the calculated value of R_P by equation (7). A equipment called Quantum Hall System (QHS), not explored in this paper, is responsible for carrying out this comparison. Its working principle consists of submitting each of these 2 resistors to a known current and increasing the value of the current I_L that circulates in R_L until the difference in the voltage drop between the 2 resistors R_L and R_P be zero, measured from a null detector. There is also a fine tuning of I_L that is done by a QHS subsystem, called Superconducting Quantum Interference Device (SQUID) where its operating principle consists of, at temperatures below 4.2 K, increase I_L as long as there is difference in the magnetic fields generated by I_L and I_P when circulating through turns with N_S and N_P windings, with 16 and 2065 turns respectively. The windings, although distinct and electrically isolated, are coupled by a magnetic circuit. The figure 8 illustrates the described operation:

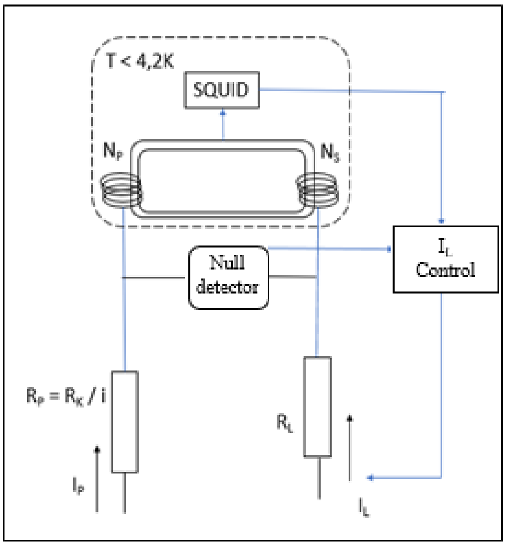

Figure 8. QHS simplified representation .

When achieved equilibrium, knowing R_P, I_P, I_L, N_P and N_S, R_L can be calculated [6].

2. Electric current primary standard based on Ohm's Law, Josephson and quantum hall effects

Consider the electric circuit proposed at the figure 9, where the current source is the device under test, R_{STD} is a resistor that has been previously calibrated by the QHS system and V_{STD} is the voltage measurement performed directly by the PJVS system:

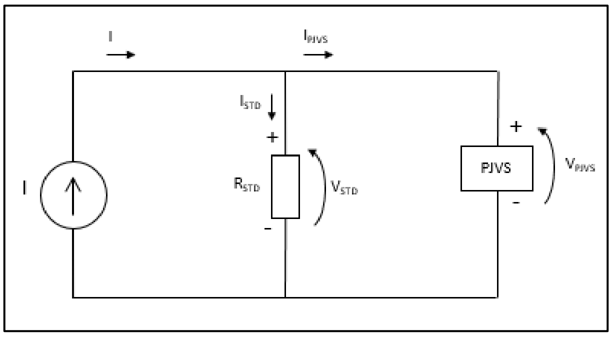

Figure 9. Circuit used to calibrate the current source I from Ohm's law.

Applying Kirchhoff's voltage law to the R_{STD} loop, the calculated I_{STD} value is given by the equation (8):

$$I_{STD} = \frac{V_{STD}}{R_{STD}} \tag{8}$$

Where:

I_{STD} is the current flowing through R_{STD};
R_{STD} is the standard resistor, directly calibrated by the QHS system;
V_{STD} is the voltage measured from the PJVS system;

Thus, the calibration of the current source I is carried out by comparing the value presented by this current source and the calculated value I_{STD}. The I_{STD} is obtained by applying equation (8), which uses V_{STD} read values (measured by the PJVS system) and R_{STD} values, R_{STD} informed by the user and with traceability to the QHS system.

In order to implement this proposal, a Fluke 5730A high performance multifunction calibrator was used as a device under test (DUT). A control software was developed in python language which commands the DUT through a GPIB interface, so that it supplies the currents to be calibrated. This same software receives from user the standard resistor information (R_{STD}) and obtains the V_{STD} value by reading the spreadsheet where the voltage measurement was recorded by the PJVS system. Therefore, in the future, this program will be added the function of calculating the current that circulates through R_{STD} (based on the model presented in equation 8) and its associated uncertainty.

The user can register the available standard resistors as well as the calibrators (DUT) that will be used by the system in the developed software. That allow the software to inform the user which are the measurement ranges that can be obtained in the DUT calibration for a selected standard resistor. After the user chooses the resistor, the software automatically distributes 5 current measurement points in each range and if the last range cannot be calibrated at its maximum value, the software automatically distributes 5 points to the maximum possible current for the last range. Additionally, in order to maintain the integrity of the standard resistor that will be used, the software does not supply the standard resistor with a current greater than the maximum recommended by the resistor manufacturer and it warns the user of this condition, if it occurs.

3. Conclusion

The PJVS and QHS systems have an operating principle based on quantum phenomena and are used as a primary voltage and electrical resistance standard, respectively.

The redefinition of the SI establishes that a unit can be realized through the measurement of quantities that do not involve the same unit and the application of the laws of physics that relate the quantity that is intended to be measured with the defining constants. Thus, it is possible to develop a primary direct current standard from existing PJVS and QHS systems, directly applying Ohm's law.

The primary standard of direct current from Ohm's law under development will be able to calibrate current sources by voltage measurement, made by the PJVS system, on standard resistors with traceability from the QHS system. The program developed is already capable of controlling the Keysight 5730A calibrator (DUT), receive from users the register of standard resistors and current sources, selecting the current values that can be calibrated for each selected resistor and storing the voltage values provided by the PJVS system so that, in the near future, be able to calculate the current and its associated uncertainty based on the model presented in equation (8).

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